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Name: ____________

Class: 12MTX______

Teacher: ____________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2006 AP4

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS EXTENSION 1

Time allowed - 2 HOURS
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- All questions are of equal value.
- Each question is to be commenced on a new page clearly marked Question 1, Question 2, etc on the top of the page. **
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- Your solutions will be collected in one bundle stapled in the top left corner.

Please arrange them in order, Q1 to 7.

Each page must show your name and your class. *

QUESTION 1 (12 marks) Start a new page **Marks**

(a) Evaluate $\lim_{x \to 0} \frac{2\sin 2x}{x}$. 2

(b) The polynomial $P(x) = 2x^3 + ax^2 + x + 2$ has a factor $(2x+1)$. Find the value of a. 2

(c) Find $\int 2\sin^2 4x \, dx$ 2

(d) Using the substitution $u = x + 2$, find $\int \frac{x}{3}\sqrt{x+2} \, dx$. 2

(e) Divide the interval *AB* externally in the ratio $4:3$, where *A* is the point $(2,-1)$ and *B* is $(1,-3)$. 2

(f) Find the obtuse angle between the lines $3x - y + 5 = 0$ and $2x + 3y - 1 = 0$. Give your answer correct to the nearest degree 2

QUESTION 2 (12 marks) Start a new page

(a) Differentiate: $\cos(\sin^{-1} 4x)$ 2

(b) For the function, $y = 3\cos^{-1}\frac{x}{2}$

(i) State the domain. 1

(ii) State the range. 1

(iii) Sketch the curve. 1

(c) Evaluate: $\sin\left[2\tan^{-1}(1)\right]$ 1

(d) (i) Show that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ 2

(ii) Given $\sin^{-1}(-\frac{2}{3}) - \cos^{-1}(-\frac{2}{3}) = k$. 4

By starting with expressions for $\sin^{-1}(-x)$ and $\cos^{-1}(-x)$, and using the result from part (i), find an expression for $\cos^{-1}(\frac{2}{3})$ in terms of k.

QUESTION 3 (12 marks) Start a new page **Marks**

(a) Use the method of mathematical induction to prove that **3**

$(1+1)+(2+3)+(3+5)+\dots+(n+(2n-1))=\frac{1}{2}n(3n+1)$

where n is a positive integer

(b) (i) Express $\sqrt{3}\cos x-\sin x$ in the form $R\cos(x+\alpha)$ where **2**

$0<\alpha<\frac{\pi}{2}$ and $R>0$.

(ii) Hence, solve $\sqrt{3}\cos x-\sin x=1$ for $0\le x\le\frac{\pi}{2}$. **1**

(c) Prove $\frac{\sin 4\theta}{\cos^2\theta-\sin^2\theta}=4\sin\theta\cos\theta$ **2**

(d) An ice cube tray is filled with water at a temperature of $18^\circ C$ and placed in a freezer that has a constant temperature of $-19^\circ C$. The cooling rate is proportional to the difference between the temperature of the freezer and the temperature of the water, T.

That is, T satisfies the equations

$$\frac{dT}{dt}=-k(T+19) \quad and \quad T=-19+Ae^{-kt}$$

(i) Show that A=37. **1**

(ii) After 5 minutes in the freezer the temperature of the water is $3^\circ C$. Find the time for the water to reach $-18.9^\circ C$. Answer correct to the nearest minute. **3**

QUESTION 4 (12 marks) Start a new page **Marks**

(a) A particle moves along the x axis. The velocity (v m/s) of the particle is described by $v=\cos^2 t$ where t is the time in seconds and x metres is the displacement from the origin O.

If $x=\frac{\pi}{4}$ when $t=\pi$, find x when $t=\frac{\pi}{2}$. **3**

(b) (i) Prove $\frac{d}{dx}(\frac{1}{2}v^2)=\frac{d^2x}{dt^2}$ **1**

(ii) The speed, v cm/s, of a particle moving along the x $axis$ is given by $v^2=72-12x-4x^2$. **2**

Show that the motion is simple harmonic.

(iii) Find the period and amplitude of the motion. **2**

(c) If $y=\frac{(2x+1)^2}{4x(1-x)}$

(i) Show that the curve $y=\frac{(2x+1)^2}{4x(1-x)}$ has three asymptotes. **2**

(ii) Given the curve has a relative maximum at $\left(-\frac{1}{2},0\right)$ and a relative minimum at $\left(\frac{1}{4},3\right)$, sketch the curve showing the asymptotes and turning points. **2**

QUESTION 5 (12 marks) Start a new page **Marks**

(a) (i) Solve the inequality $\frac{1}{1-x}<1$ 2

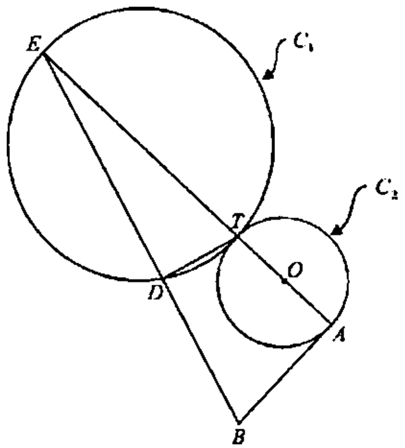
(ii) Hence find the set of values of x for which the limiting sum S of the geometric series $1+x+x^2+x^3+\ldots$ is such that $S<1$ 1

(b) (i) Show that the equation of the normal at $P(at^2,2at)$ on the parabola $y^2=4ax$ is $y+tx=at^3+2at$ 2

(ii) The normal intersects with the x-axis at point Q. Find the coordinates of Q and hence show that the coordinates of R, the midpoint of PQ is $(a(1+t^2),at)$. 2

(iii) Hence find the Cartesian equation of the locus of R. 1

(c)



NOT TO SCALE

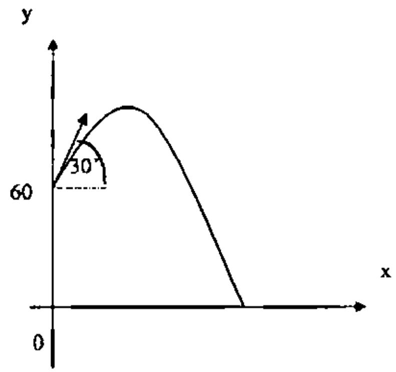
Two circles C_1 and C_2 touch at T. The line AE passes through O; the centre of C_2, and through T. The point A lies on C_2 and E lies on C_1. The line AB is a tangent to C_2 at A, D lies on C_1 and BE passes through D. The radius of C_1 is R and the radius of C_2 is r.

(i) Show that $\angle EDT=90^\circ$, giving reasons for your answer. 1

(ii) If $DE=2r$ find an expression for the length of EB in terms of r and R. 3

QUESTION 6 (12 marks) Start a new page **Marks**

(a) A ball is projected from the top of a 60m vertical cliff with a velocity of 10 m/s at an angle of 30° above the horizontal. Take the origin as (0,0). Assume $g=10\,m/s^2$.



(i) Show using calculus that $x=5\sqrt{3}t$ and $y=-5t^2+5t+60$ 3

(ii) Find the maximum height of the ball above the ground. 2

(iii) Find the time that elapses before the ball hits the ground 1

(iv) Find the Cartesian equation of the trajectory of the ball. 2

(b) (i) Show that the equation $x^3+2x-7=0$ has a root α such that $1<\alpha<2$. 2

(ii) If an initial approximation of $1\cdot5$ is taken for α, use one application of Newton's method to find the next approximation, rounding your answer to one decimal place. 2

QUESTION 7 (12 marks) Start a new page **Marks**

(a) Give the general solution to $(2\sin\theta+1)(\cos\theta-1)=0$. Answer in terms of π. 2

(b) The volume, $V\,m^3$, of usable wood in a tree of radius R metres can be modelled using the formula

$$\log_e V = 3\log_e 2R - 0\cdot 81$$

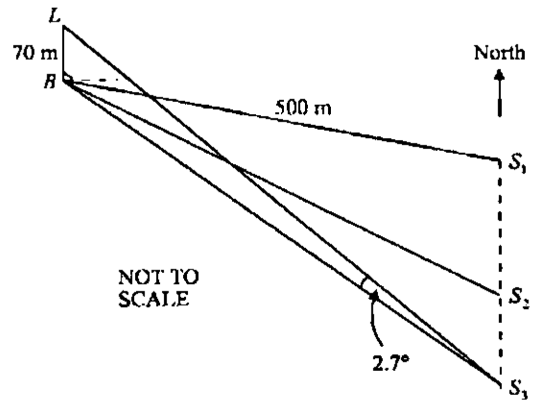
(i) Using the approximation $e^{0\cdot 81} \approx 2\cdot 25$ show that the formula $\log_e V = 3\log_e 2R - 0\cdot 81$ can be expressed as $V \approx \frac{32R^3}{9}$. 3

(iii) The radius of the tree is increasing at a rate of $0\cdot 002$ metres/year. At what rate is the usable volume of the wood in the tree increasing when the radius of the tree is $1\cdot 2\,m$? (Answer in $m^3/year$ to 2 significant figures) 2

Question 7 is continued on next page

(c) A life saver sits at point L, 70 m above sea level, in a lookout on a cliff. He spots a surfer lying on his board at point S_1. S_1 is on a bearing of 120° in relation to the life savers position, and a distance of 500 m from point B which is located at sea level directly below the life saver.

Twenty minutes later the life saver notes that the surfer is still lying on his surfboard but is now located at point S_2 on a bearing of 160° in relation to the life savers position and has moved in a line due south.



The life saver raises the alarm and twenty-three minutes after his initial sighting a rescue boat is launched from point B. From point S_3, which is the point at which the rescue boat reaches the surfer, the angle of elevation of the life saver in the tower is $2\cdot 7°$.

Assume that the rescue boat has travelled at a constant speed and in a straight line to reach the surfer and that the surfer has drifted at a constant speed in a direction due south since the initial sighting.

(i) Show that the distance $S_1S_2 = 250\sqrt{3}\tan 70° - 250$ metres 1

(ii) Show that the speed at which the surfer was drifting south was $2\cdot 82$ km/h (correct to 2 decimal places). 1

(iii) Find the speed at which the rescue boat was moving. Express your answer in km/h correct to 2 decimal places. 3

END OF TEST

Question 1

a) $\lim_{x\to 0} \frac{2\sin 2x}{x} = \lim_{x\to 0} \frac{4\sin 2x}{2x}$ ①

$= 4\times 1$

$= 4$ ①

b) $P(x) = 2x^3 + ax^2 + x + 2$

$P(-\frac{1}{2}) = 0$ since $(2x+1)$ is a factor

$P(-\frac{1}{2}) = 2(-\frac{1}{2})^3 + a(-\frac{1}{2})^2 + (-\frac{1}{2}) + 2$

$0 = -\frac{1}{4} + \frac{a}{4} - \frac{1}{2} + 2$ ①

$-\frac{5}{4} = \frac{a}{4}$

$a = -5$ ①

c) $\int 2\sin^2 4x\,dx = 2\times\frac{1}{2}\int(1-\cos 8x)\,dx$ ①

$= x - \frac{1}{8}\sin 8x + c$ ①

(ignore c)

d) $u = x+2$, $\frac{du}{dx} = 1$, $x = u-2$

$\int \frac{x}{3}(\sqrt{x+2})\,dx$

$= \int \frac{u-2}{3}(u^{1/2})\,du$

$= \frac{1}{3}\int u^{3/2} - 2u^{1/2}\,du$ ①

$= \frac{1}{3}\left[\frac{2u^{5/2}}{5} - \frac{4u^{3/2}}{3}\right] + C$

$= \frac{1}{3}\left(\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2}\right] + C$ ①

$= \frac{2}{15}\sqrt{(x+2)^5} - \frac{4}{9}\sqrt{(x+2)^3} + C$

e) A $(2,-1)$ B $(1,-3)$ $4:-3$

$\left(\frac{4(1)-3(2)}{4-3}, \frac{4(-3)-3(-1)}{4-3}\right)$ ①

$= (-2,-9)$ ①

f) $m_1 = 3$, $m_2 = \frac{-2}{3}$

$\tan\theta = \left|\frac{3+\frac{2}{3}}{1+3(\frac{-2}{3})}\right|$ ①

$= |-3\frac{2}{3}|$

$\theta = 75^\circ$

but $\theta = 105^\circ$ for obtuse ①

Alternative — Need diagram, Need exterior $\angle$ of Δ theorem

Angle of inclination of $3x - y + 5 = 0$

$\theta_1 = \tan^{-1}3 = 71^\circ 34'$

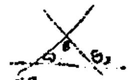
Angle of inclination of $2x + 3y - 1 = 0$

$\theta_2 = \tan^{-1}(\frac{-2}{3}) = 146^\circ 19'$ ①

$\theta = 146^\circ 19' - 71^\circ 34'$ (ext. $\angle$ of Δ)

$= 74^\circ 45'$ ①

$\therefore$ obtuse $\angle = 180 - 75^\circ = 105^\circ$



Question 2

a) $\frac{d}{dx}\cos(\sin^{-1}4x) = \frac{4}{\sqrt{1-16x^2}}\times -\sin(\sin^{-1}4x)$ ①

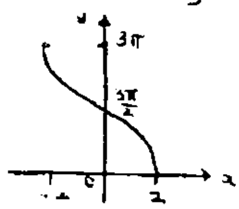
$= \frac{-16x}{\sqrt{1-16x^2}}$ ①

b) $y = 3\cos^{-1}\frac{x}{2}$

i) Domain $-1 \le \frac{x}{2} \le 1 \Rightarrow -2 \le x \le 2$ ①

ii) Range $0 \le y \le 3\pi$ ①

iii)



①

c) $\sin\left[2\tan^{-1}(1)\right] = \sin(2\times\frac{\pi}{4})$

$= \sin\frac{\pi}{2}$

$= 1$ ①

d) i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

Method 1: let $y = \sin^{-1}x$

$x = \sin y$ ①

so $x = \cos(\frac{\pi}{2} - y)$

$\therefore \frac{\pi}{2} - y = \cos^{-1}x$ ①

LHS $= \sin^{-1}x + \cos^{-1}x$

$= y + \frac{\pi}{2} - y$

$= \frac{\pi}{2}$

$=$ RHS.

Method 2: $\frac{d}{dx}(\sin^{-1}x + \cos^{-1}x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}$

$= 0$ ①

$\therefore \sin^{-1}x + \cos^{-1}x = c$ (constant)

when $x = 1$: $\sin^{-1}(1) + \cos^{-1}(1) = \frac{\pi}{2} + 0 = \frac{\pi}{2}$

$\therefore c = \frac{\pi}{2}$ ①

$\therefore \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

d) $\sin^{-1}(-x) = -\sin^{-1}x$, $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ ①

$\sin^{-1}(-\frac{2}{3}) = -\sin^{-1}(\frac{2}{3})$ and $\cos^{-1}(-\frac{2}{3}) = \pi - \cos^{-1}(\frac{2}{3})$ ①

Now $\sin^{-1}(-\frac{2}{3}) - \cos^{-1}(-\frac{2}{3}) = k$

$-\sin^{-1}(\frac{2}{3}) - \pi + \cos^{-1}(\frac{2}{3}) = k$

$\cos^{-1}(\frac{2}{3}) = k + \pi + \sin^{-1}(\frac{2}{3})$ ①

From (1): $\sin^{-1}(\frac{2}{3}) = \frac{\pi}{2} - \cos^{-1}(\frac{2}{3})$

$\cos^{-1}(\frac{2}{3}) = k + \pi + \frac{\pi}{2} - \cos^{-1}(\frac{2}{3})$

$2\cos^{-1}(\frac{2}{3}) = k + \frac{3\pi}{2}$

$\cos^{-1}(\frac{2}{3}) = \frac{2k + 3\pi}{4}$ ①

Question 3

a) $(1+1)+(2+3)+(3+5)+\dots+(n+(2n-1)) = \frac{1}{2}n(3n+1)$

Step 1 : prove $n=1$

LHS $= 1+1$
$= 2$

RHS $= \frac{1}{2}(1)(3\times1+1)$
$= \frac{1}{2}\times 4$
$= 2$

$\therefore$ LHS = RHS ①

$\therefore$ true for $n=1$.

Step 2 : assume true for $n=k$

$(1+1)+(2+3)+(3+5)+\dots+(k+(2k-1)) = \frac{1}{2}k(3k+1)$

Step 3 : Prove true for $n=k+1$

$(1+1)+(2+3)+(3+5)+\dots+((k+1)+(2(k+1)-1)) = \frac{1}{2}(k+1)(3k+4)$ ①

LHS $= \underbrace{(1+1)+(2+3)+(3+5)+\dots+(k+(2k-1))}_{\text{from assumption}} + ((k+1)+(2k+1))$

$= \frac{1}{2}k(3k+1) + (k+1) + (2k+1)$
$= \frac{3k^2}{2} + \frac{k}{2} + k+1+2k+1$
$= \frac{3k^2}{2} + \frac{7k}{2} + 2$
$= \frac{1}{2}(3k^2+7k+4)$
$= \frac{1}{2}(k+1)(3k+4)$
= RHS. ①

$\therefore$ since prove true for $n=k+1$ when assumed true for $n=k$ and prove true for $n=1$, it follows it must be true for all integral values of n.

b) i) Method 1

$R\cos(x+\alpha) = R\cos x\cos\alpha - R\sin x\sin\alpha$

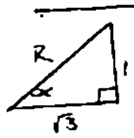
$\sqrt{3}\cos x - \sin x = R\cos(x+\alpha)$
$= R(\cos x\cos\alpha - \sin x\sin\alpha)$

$R\cos\alpha = \sqrt{3}$
$R\sin\alpha = 1$
$\tan\alpha = \frac{1}{\sqrt{3}}$ $\alpha = \frac{\pi}{6}$ ① for either R or α.

$R = \sqrt{(\sqrt{3})^2+1^2} = 2.$

$\therefore \sqrt{3}\cos x - \sin x = 2\cos\left(x+\frac{\pi}{6}\right)$ ①

Method 2



$\cos\alpha = \frac{\sqrt{3}}{R}$ $\sin\alpha = \frac{1}{R}$ $\tan\alpha = \frac{1}{\sqrt{3}}$

where $R = \sqrt{(\sqrt{3})^2+1^2} = 2$ $\alpha = \frac{\pi}{6}$ ①

$\sqrt{3}\cos x - \sin x = 2\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right)$
$= 2(\cos x\cos\alpha - \sin x\sin\alpha)$
$= 2\cos(x+\alpha)$
$= 2\cos\left(x+\frac{\pi}{6}\right)$ ①

(ii) $2\cos\left(x+\frac{\pi}{6}\right) = 1$

$\cos\left(x+\frac{\pi}{6}\right) = \frac{1}{2}$

$x+\frac{\pi}{6} = \frac{\pi}{3}$

$x = \frac{\pi}{6}$ ①

c) $\frac{\sin 4\theta}{\cos^2\theta - \sin^2\theta} = 4\sin\theta\cos\theta$

LHS $= \frac{\sin 4\theta}{\cos^2\theta - \sin^2\theta}$
$= \frac{2\sin 2\theta\cos 2\theta}{\cos 2\theta}$ ①
$= 2\sin 2\theta$
$= 2\times 2\sin\theta\cos\theta$ ①
$= 4\sin\theta\cos\theta$
= RHS.

* a correct proof aw 2

* using $\sin 2\theta$ or $\cos 2\theta$ results correctly aw ①

d) i) when $t=0$ $T=18$

$18 = -19 + Ae^{-k\times 0}$ ← must have this line

$A = 19+18$ ①

$\therefore A = 37.$

ii) $t=5$ $T=3$

$3 = -19 + 37e^{-5k}$ ①

$e^{-5k} = \frac{22}{37}$

$k = -\frac{1}{5}\log_e\left(\frac{22}{37}\right)$ ①

$= 0.103975091$

(iv)

When $T = -18.9$

$-18.9 = -19 + 37e^{-kt}$

$37e^{-kt} = 0.1$

$e^{-kt} = \frac{0.1}{37}$ $\left(= \frac{1}{370}\right)$

$-kt = \log_e\left(\frac{0.1}{37}\right)$

$t = \frac{\log_e\left(\frac{0.1}{37}\right)}{-k}$

$t = 56.87422728$

$\therefore t = 57$ minutes ①

Question 4

a) $v = \cos^2 t$

$x = \int \cos^2 t \, dt$

$= \frac{1}{2}\int 1 + \cos 2t \, dt$

$x = \frac{1}{2}\left(t + \frac{\sin 2t}{2}\right) + C$ ①

$x = \frac{\pi}{4}$ when $t = \pi$

$\frac{\pi}{4} = \frac{1}{2}\left(\pi + \frac{\sin 2\pi}{2}\right) + C$

$C = \frac{\pi}{4} - \frac{\pi}{2}$

$C = -\frac{\pi}{4}$

$\therefore x = \frac{1}{2}\left(t + \frac{\sin 2t}{2}\right) - \frac{\pi}{4}$ ①

When $t = \frac{\pi}{2}$

$x = \frac{1}{2}\left(\frac{\pi}{2} + \frac{\sin \pi}{2}\right) - \frac{\pi}{4}$

$= \frac{\pi}{4} - \frac{\pi}{4}$

$\therefore x = 0$ ①

b) i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dv}\left(\frac{1}{2}v^2\right) \times \frac{dv}{dx}$

$= v \times \frac{dv}{dx}$

$= \frac{dx}{dt} \times \frac{dv}{dx}$

$= \frac{dv}{dt}$

$= \frac{d^2x}{dt^2}$

correct method. ①

(ii) $v^2 = 72 - 12x - 4x^2$

$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left(36 - 6x - 2x^2\right)$ ①

$= -6 - 4x$

$\ddot{x} = -4\left(x + \frac{3}{2}\right)$ ①

$\therefore$ It is in simple harmonic motion as form is $\ddot{x} = -n^2 x$.

(iii) $n = 2 \to$ period $= \frac{2\pi}{2} = \pi$ ①

It stops when $v^2 = 0$ ie $4x^2 + 12x - 72 = 0$

$4(x - 3)(x + 6) = 0$

$\therefore$ It stops at $x = 3$ and $x = -6$.

with centre of motion at $-\frac{3}{2}$

$\therefore$ amplitude $= 4\frac{1}{2}$ units ①

Q4 continued

c)(i) $y = \frac{(2x+1)^2}{4x(1-x)}$

since denominator is undefined for $x = 0$ and $x = 1$ then vertical asymptotes occur at 0 and 1. ①

<u>method 1:</u> $\lim_{x \to \infty} \frac{(2x+1)^2}{4x(1-x)} = \lim_{x \to \infty} \frac{4x^2 + 4x + 1}{4x - 4x^2}$

$= \lim_{x \to \infty} \frac{\frac{4x^2}{x^2} + \frac{4x}{x^2} + \frac{1}{x^2}}{\frac{4x}{x^2} - \frac{4x^2}{x^2}}$

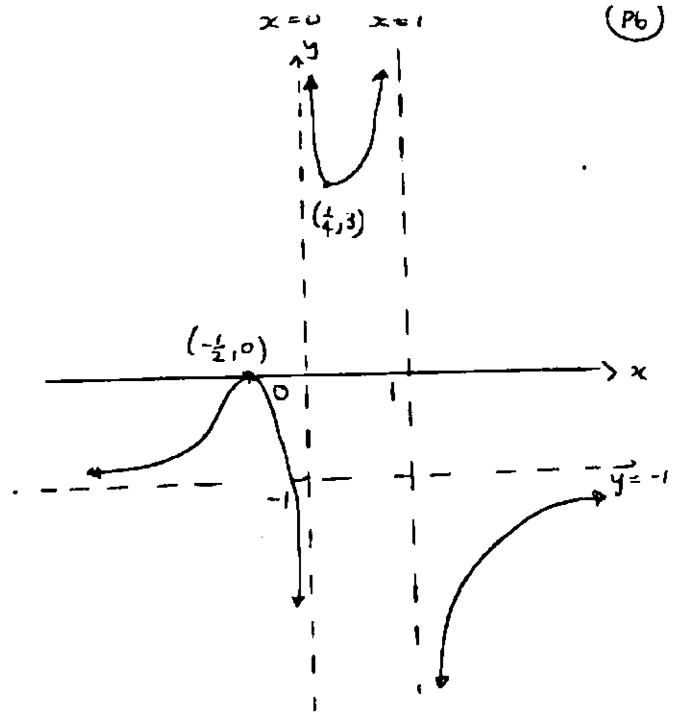
$= \lim_{x \to \infty} \frac{4 + \frac{4}{x} + \frac{1}{x^2}}{\frac{4}{x} - 4}$ ①

$= -1$ (as $\frac{4}{x}, \frac{1}{x^2} \to 0$ as $x \to \infty$)

$\therefore$ horizontal asymptote at -1.

$\therefore$ asymptotes occur at $x = 0$, $x = 1$ and $y = -1$.

(ii)



① shape

① asymptotes + stationary points

Question 5

a) i) $\frac{1}{1-x} < 1$

$\frac{(1-x)^2}{1-x} < (1-x)^2$

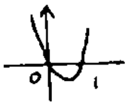
$1-x < (1-x)^2$ ①

$(1-x)^2 - (1-x) > 0$

$(1-x)(1-x-1) > 0$

$-x(1-x) > 0$

ie $x(x-1) > 0$



$x < 0$ or $x > 1$ ① (Ignore "or" but cross out "and")

(ii) Limiting sum $S = \frac{1}{1-x}$

but $S < 1$ and $|x| < 1$

so $\frac{1}{1-x} < 1$ and $-1 < x < 1$

$\therefore x < 0$ or $x > 1$

$\therefore$ solution is $-1 < x < 0$ ①

b) i) $x = at^2$ $y = 2at$

$\frac{dx}{dt} = 2at$ $\frac{dy}{dt} = 2a$

$\frac{dy}{dx} = \frac{dt}{dx} \times \frac{dy}{dt}$

$= \frac{1}{2at} \times 2a$

$= \frac{1}{t}$

$\therefore m_T = \frac{1}{t}$ $m_N = -t$ ① (since $m_1 m_2 = -1$)

Equation of normal

$y - 2at = -t(x - at^2)$

$y - 2at = -tx + at^3$

$y + tx = at^3 + 2at$ ①

(ii) Point Q has $y = 0$

$tx = at^3 + 2at$

$x = at^2 + 2a$

$Q\ (at^2 + 2a,\ 0)$ ①

Midpoint PQ $R = \left[\frac{at^2 + at^2 + 2a}{2}, \frac{2at + 0}{2}\right]$

$= (a + at^2,\ at)$

$= (a(1+t^2),\ at)$ ①

Q5 continued

b) (iii) At R: $x = a(1+t^2)$

$y = at \rightarrow \frac{y}{a} = t$

$\therefore x = a\left(1 + \frac{y^2}{a^2}\right)$

$\frac{x}{a} = 1 + \frac{y^2}{a^2}$

$y^2 = a^2\left(\frac{x}{a} - 1\right)$

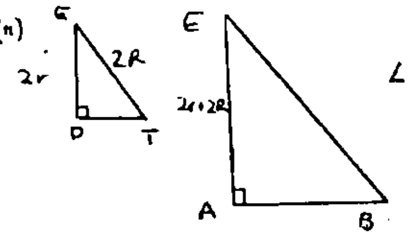
$y^2 = a(x - a)$ ①

c) i) Since C_1 and C_2 touch at T and AT is a diameter of C_2 then AG passes through the centre of C_1 (theorem: when circles touch, line through centre passes through point of contact).

$\therefore$ ET is a diameter.

$\therefore \hat{EDT} = 90°$ ① ($\angle$ in semi-circle is a right $\angle$).

(ii)



$\angle EAB = 90°$ (tangent to circle is ⊥ to radius at point of contact) ①

In Δ^s ABE and DTE

$\hat{EDT} = \hat{EAB} = 90°$ (part i)

$\hat{DET}$ is common.

$\therefore \Delta ABE \parallel\!| \Delta DTE$ (equiangular) ①

$\frac{EB}{ET} = \frac{EA}{ED}$

$\frac{EB}{2R} = \frac{2r + 2R}{2r}$

$EB = \frac{2R(2r + 2R)}{2r} = \frac{2R(r+R)}{r}$ ①

Alternative

Let $\hat{DET} = \theta$ then in ΔDET

$\cos\theta = \frac{2r}{2R}$ ①

In ΔAEB

$\hat{EAB} = 90°$ (tangent .....). ①

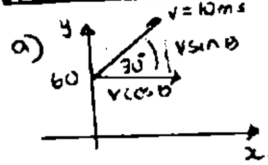
$\cos\theta = \frac{2r + 2R}{EB}$

$EB = \frac{(2r + 2R)}{\cos\theta} = \frac{(2r + 2R)}{r/R}$

$= \frac{2R(r+R)}{r}$ ①

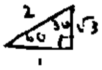
Question 6

a)



$g = 10\,ms^{-2}$

$v = 10$



(i) $\ddot{x} = 0$

$\dot{x} = C$

when $t = 0$ $\dot{x} = 10\cos 30$

$\dot{x} = 10\cos 30$

$= 10 \times \frac{\sqrt{3}}{2}$

$\dot{x} = 5\sqrt{3}$

$x = 5\sqrt{3}t + C$

when $t = 0$ $x = 0$

$\therefore C = 0$

$\therefore x = 5\sqrt{3}t$ (1)

$\ddot{y} = -10$

$\dot{y} = -10t + c$

when $t = 0$ $\dot{y} = 10\sin 30$

$\therefore c = 10\sin 30 = 10 \times \frac{1}{2} = 5$

$\dot{y} = -10t + 5$ (1)

$y = -5t^2 + 5t + c$

when $t = 0$ $y = 60$

$60 = c$ (1)

$\therefore y = -5t^2 + 5t + 60$

(ii) maximum height occurs when $\dot{y} = 0$

$-10t + 5 = 0$

$t = \frac{1}{2}$ second (1)

$y = -5\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + 60 = -\frac{5}{4} + \frac{5}{2} + 60$

$= 61\frac{1}{4}$ metres (1)

(iii) when $y = 0$ $0 = -5t^2 + 5t + 60$

$t^2 - t - 12 = 0$

$(t - 4)(t + 3) = 0$

but $t \geq 0$ $\therefore$ $t = 4$ seconds (1)

(iv) $t = \frac{x}{5\sqrt{3}}$ (1) $y = -5\left(\frac{x}{5\sqrt{3}}\right)^2 + 5\left(\frac{x}{5\sqrt{3}}\right) + 60$

$y = -\frac{x^2}{15} + \frac{x}{\sqrt{3}} + 60$ (1)

(b) (i) Let $f(x) = x^3 + 2x - 7$

$f(1) = 1^3 + 2(1) - 7 = -4 < 0$
$f(2) = 2^3 + 4 - 7 = 5 > 0$] (1)

since $f(x)$ is a continuous function of x there exists a root $\alpha \rightarrow 1 < \alpha < 2$ such that $f(\alpha) = 0$ (1)

(ii) $f(x) = x^3 + 2x - 7$

$f'(x) = 3x + 2$

$\alpha_1 = 1.5 - \frac{f(1.5)}{f'(1.5)}$ (1)

$= 1.5 - \frac{(1.5^3 + 2(1.5) - 7)}{3(1.5) + 2}$

$= 1.571428571$
$\alpha = 1.6$ (to 1 decimal place)] (1)

Question 7

a) $(2\sin\theta + 1)(\cos\theta - 1) = 0$

$\sin\theta = -\frac{1}{2}$ $\cos\theta = 1$

$\theta = n\pi + (-1)^n \sin^{-1}\left(-\frac{1}{2}\right)$

$= n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$
$= n\pi + (-1)^{n+1}\left(\frac{\pi}{6}\right)$] (1)

$\theta = 2n\pi$ (1)

b) i) $e^{0.8t} = 2.25$

$\log_e e^{0.8t} = \log_e 2.25$

$0.8t = \log_e 2.25$ (1)

$\log_e V = 3\log_e 2R - 0.81$

$\log_e V = \log_e (2R)^3 - \log_e 2.25$

$\log_e V = \log_e \frac{8R^3}{2.25}$ (1)

$V = \frac{8R^3}{2.25} \times \frac{4}{4}$ (1)

$V = \frac{32R^3}{9}$

(ii) $\frac{dR}{dt} = 0.002\,m/\text{year}$

$V = \frac{32R^3}{9}$

$\frac{dV}{dR} = \frac{32R^2}{3}$

$\frac{dV}{dt} = \frac{dV}{dR} \times \frac{dR}{dt}$

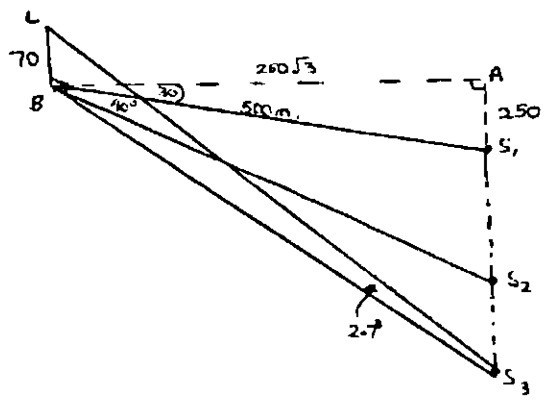
$= \frac{32R^2}{3} \times 0.002$ (1)

when $R = 1.2$

$\frac{dV}{dt} = \frac{32(1.2)^2 \times 0.002}{3}$

$= 0.03072$
$= 0.031\,m^3/\text{year}$] (1)

c)



(i) In ΔABS_1 $AB = 500\cos 30 = 250\sqrt{3}$

$AS_1 = 500\sin 30 = 250$ ①

In ΔABS_2 $AS_2 = 250\sqrt{3}\tan 70$

so $S_1S_2 = 250\sqrt{3}\tan 70 - 250$

$= 939.6926208$

(ii)

Speed of surfer drifting south $= \dfrac{\text{distance } S_1S_2}{20\text{mins}} = \dfrac{250\sqrt{3}\tan 70 - 250\text{m}}{20\text{min}}$

$= \dfrac{0.9396926208\text{ km}}{\frac{1}{3}\text{h}}$

$= 2.819077862$ ①

$= 2.82$ km/h.

(c) on next page.

Q7 continued

c)(iii) In ΔBLS_3 $BS_3 = \dfrac{70}{\tan 2.7}$

$= 1484.346415$

In ΔABS_3 $AS_3 = \sqrt{(BS_3)^2 - AB^2}$

$= \sqrt{\left(\frac{70}{\tan 2.7}\right)^2 - (250\sqrt{3})^2}$

$= 1419.783181$

$\therefore S_2S_3 = AS_3 - S_1S_2 - AS_1$

$= 1419.783181 - 939.6926208 - 250$

$= 230.09056$ m ①

Time taken for surfer to drift from S_2 to S_3

$= \dfrac{0.2309056\text{ km}}{2.82\text{ km/h}}$ or $= \dfrac{0.2309056}{2.819077862}$

$= 0.081592397$ h ① $= 0.081908202$ h.

*The rescue boat leaves 3 minutes (0.05h) after lifesaver spots the surfer at S_2.

$\therefore$ rescue boat takes $= 0.081592397 - 0.05$

$= 0.031592397$ h

OR $= 0.081908202 - 0.05$

$= 0.031908202$ h

• Speed of rescue boat $= \dfrac{BS_3}{\text{time taken}}$

$= \dfrac{1.484346415\text{km}}{0.031592397\text{ h}} = 46.9842923$ km/h.

$= 46.98$ km/h

or $= \dfrac{1.484346415\text{ km}}{0.031908202\text{ h}} = 46.51927473$ km/h.

$= 46.52$ km/h ①

*Marks Aw ① S_2S_3

Aw ① time taken for surfer to drift from S_2 to S_3

Aw ① speed of rescue boat.

➡ Note there will be many variations on the answers based on where the students round off.

Comments on Yr 12 AP4 Extension Maths

Q1 a) Too many students failed to recognise $\lim_{x \to 0} \frac{\sin x}{x} = 1$

Some went the way converting $\sin 2x$ into $2 \sin x \cos x$

Some stated $\lim_{y \to 0} \frac{\sin x}{x} = \frac{\sin x}{x}$.

d) A lot of students stopped at $\frac{2}{15}u^{\frac{5}{2}} - \frac{4}{9}u^{\frac{3}{2}} + C$. Should leave the answer in terms of x.

(f) Surprisingly large no. of students
 a) failed to use the formula correctly or using wrong formula.
 b) did not give the obtuse angle.

Q3 a) The mathematical induction question: the setting out is causing concern. Too many students wrote

Let $n=k$ $\therefore (1+1)+(2+3)+\cdots+[k+(2k-1)] = \frac{1}{2}k(3k+1)$

Let $n=k+1$ $\therefore (1+1)+(2+3)+\cdots[k+1+2(k+1)-1] = \frac{1}{2}(k+1)[3(k+1)+1]$

The problem with this setting out is: once you write

something (P) = something (Q)

You have MADE a statement claiming P = Q is true !!

Student should write

Assume it is true for $n=k$

ie $(1+1)+(2+3)+\cdots+[k+(2k-1)] = \frac{1}{2}k(3k+1)$

then we test (or aim to prove true) for $n=k+1$

ie whether $(1+1)+(2+3)+\cdots+[k+(2k-1)]+[(k+1)+2(k+1)-1] = \frac{1}{2}(k+1)[3(k+1)+1]$ is true

c) A lot of students failed to recognise $\cos^2\theta - \sin^2\theta = \cos 2\theta$

Q7. a) The performance on (a) is left a lot to be desired. Students giving answer like $n\pi + (-1)^n(\frac{7\pi}{6})$ obviously failed to connect the range of $\sin^{-1}x$ is $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ to general solution of equations like $\sin\theta = -\frac{1}{2}$

b) Surprise, surprise!

Far too many students claiming $e^{a-b} = e^a - e^b$.

They wrote $\log_e V = 3\log_e 2R - 0.81$

$\therefore V = e^{3\log_e 2R} - e^{0.81}$!!!

c) This part was badly attempted, most probably due to the time factor. But student should be smart enough to pick up the 1 mark awarded to (ii) by using the result from (i).

Markers Comments Q 2, 4, 6

Q 2 (a) Poorly completed → many did not recognise
∴ $\sin(\sin^{-1} 4x) = 4x$

(b) (i) ✓, (ii) ✓

(iii) → Needed to show $\frac{3\pi}{2}$ → y intercept for mark, quite a few drew inverse cos back to front

(c) Could do on calculator → many did not attempt.

(d) (i) Quite a few fudges → PTO for alternate solution

(ii). many did not read question and write down expressions for $\sin^{-1}(-x)$ and $\cos^{-1}(-x)$ without expressions for $\sin^{-1}(-x)$ and $\cos^{-1}(-x)$ → max 3/4.

Q 4. a) Poor reading of question → poor results.

(b) (i) → many fudges.

(ii). Poorly answered.

(iii) → Needed to find Period + Amplitude from part (ii)

(c)(i) OK

(ii) OK.

Q 6 a) i) $x = 5\sqrt{3}t$ → needed to show calculation of constants

$y = -5t^2 + 5t + 60$ → same for constants.

(ii) OK (iii) OK (iv) OK.

b) (i) 1 mark for $f(1)$ $f(2)$, 2nd mark for specifying that change in sign → a root between (ii) Well done.

Q 2 continued

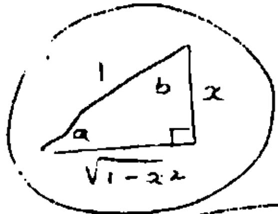
ACCEPTED

let $a = \sin^{-1} x$

$x = \sin a$

$b = \cos^{-1} x$

$x = \cos b$



(1).

but $a + b = 90 = \frac{\pi}{2}$ ($\angle$ sum of $\Delta = 180°$) (1)

$\therefore \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.

Q 4 (ii) ACCEPTED

$v^2 = n^2(a^2 - x^2)$

$v^2 = 72 - 12x - 4x^2$

$v^2 = 4(18 - 3x - x^2)$

$= 4\left(\frac{81}{4} - \left(x + \frac{3}{2}\right)^2\right)$

$= 2^2\left(\left(\frac{9}{2}\right)^2 - \left(x + \frac{3}{2}\right)^2\right)$

∴ in required form.

MARKING COMMENTS – QUESTION 5

(a) (ii) Very poorly done – very few took into account $-1 < x < 1$ for a limiting sum to exist.

(b) (i) Many students decided that $\dot{x} = t$ meant that $\frac{dy}{dx} = t$ rather than $\frac{dx}{dy}$. As a result, they used $x - at^2 = \frac{-1}{t}(y - 2at)$ FOR SOME REASON and got the correct final form. If they claimed that the gradient of tangent was t they received zero out of 2 for this part. If they didn't mention this, I had to give them 2 marks.

(ii) Change $R = (a(1+t), at)$ to $R = (a(1+t^2), a$

(c) If students found $\angle USBT = 90°$ with reason (even in part (ii)) I gave them the mark. Same for $\angle FAB = 90°$ with reason – if given in (i). Basically I marked this as a single question in many ways.